**Scenario Optimization Results**

**Scenario Optimization**

*Dembo*

* Standard linear optimization problem
* In a large part of data, we might have uncertainty dependent on future events
  + Restating the original problem as a stochastic linear problem
    - The ‘u’ subscript denotes uncertainty, the ‘d’ subscript denotes determinism
* Uncertainty can also be represented using scenarios
  + A realization/instance of uncertain data
  + Restating the stochastic linear problem
    - Let each scenario
  + The above problem is now deterministically represented using scenario subproblems
  + Let denote the solution to a particular scenario
  + Associated with each scenario *s* is a probability *Ps*that the scenario occurs
    - Often, these values will change
    - Assume the probabilities are given
* The fundamental question: How should solutions to different scenarios be combined to form a single solution to the underlying stochastic problem?
  + A solution, to the stochastic system is feasible if it satisfies the following deterministic constraints
    - is minimized
  + Assuming stochastic linear system uses the equality
    - Minimize
    - This is a tracking model
      * One out of many possible coordination models to penalize bad solutions
      * This model minimizes the norm-squared rather than the norm to make the solution easily differentiable
* The key to this approach is that the scenario subproblems and the coordination problem are decoupled
  + If all scenarios were known beforehand, the solution is easily computable
* Scenario optimization approach applies to non-linear systems as well
  + Can be generalized further
    - All that is required are input scenarios and value of inputs tracked
    - Black box conditions apply
* Scenarios and probability generation is independent and must be derived from intuition or provided otherwise

**Uncertain Convex Programs: Randomized Solutions and Confidence Levels**

*Calafiore*

* An approach to solving UCP is to randomly sample the uncertainty parameter and substitute the original infinite constraint set with a finite set of N constraints
  + Providing the samples are sufficient, only a small portion of original constraints are unsatisfied
    - The number of constraints that unsatisfied rapidly decreases as N increases
* Definition of UCP
  + *x* is the optimization value
  + *X* is a convex and closed set
    - Continuous and convex for all combination of values
  + is intended element-wise
* Robust convex optimization is unfeasible as a solution to this problem as it lifts the class of the problem and makes it computationally difficult
* An alternate approach is to randomize the uncertainty parameter (delta)
  + Assume the uncertain problem family is parametrized by instance parametres of δ
  + By drawing N samples of the instance parameter, the infinite constraint set of RCP is substituted with a finite set of N
    - The equivalent of solving a standard convex problem with N constraints
* How many samples need to be drawn to guarantee that the randomized solution violates only a small portion of the constraints?
  + Assume that is endowed with σ-algebra D
  + Probability measure P over D is assigned
    - P can be the probability that δ takes a particular value
    - Can also be the relative importance attributed to different instances
    - Assume a uniform probability density for this instance
  + A solution is e-level feasible if the probability of violating any constraint is *e*
  + The number of samples required is
    - n is the size of the solution vector
    - e is the error level
    - B is the confidence level
      * 1-B refers to the probability PN of extracting a bad multisample such that the solution does not meet e-level feasibility
    - **This bound is made logarithmic in a later paper by Calafiore**
  + Remarks
    - Regardless of P, the above bound applies
    - It is guaranteed that the solution will be e-level feasible with probability not smaller than 1-B
    - If multiple solutions exist for the problem, apply an arbitrary selection procedure
    - If no solutions exist, it may be because the simplified problem is unbounded, in which case, a resampling must take place
    - If a small risk of failure is accepted, UC can be solved efficiently with a randomized algorithm irrespective of parametres

**Scenario Approach to Robust Control Design**

*Calafiore*

* Same premise as the last paper
* The sample complexity bound is refined to the following logarithmic bound
  + N is the sample size
  + e is the error level
  + n is the dimension of the solution vector/space
  + B is the confidence level
* This is comparable to the bounds established by Occam and PAC

**Sampling and Discarding Approach to Chance Constrained Optimization**

*Campi*

* Chance Constrained Program
  + is a family of constraints parametrized by (uncertain parameter)
  + Let be endowed with σ-algebra D, distributed by P.
    - With the uncertainty, the probability of the constraint in the set must be greater than 1-e
  + In CCP, constraint violation is tolerated, but must be no larger than e
* This paper considers a sample based approximation, replacing with a finite number (N) of sample instances of , distributed by P.
  + Furthermore, samples are also discarded in order to optimize performance
    - The number discarded is k
  + The objective is to relate sample based approximation to the initial CCP problem and provide sample size results on number of constraints to get a feasible solution
* An assumption being made for the results in this paper is that the sample removal algorithm is optimal
  + Disregard the cost of an optimal algorithm
  + Several other algorithms exist (i.e. greedy)
  + The probability of violation increases linearly (approximately), with the number of samples removed
    - However, the cost exponentially decreases
  + The further assumption being made is that active constraints are eliminated
    - In doing so, the new solution x\* definitely violates the removed constraints
* Definition: Violation probability of a given x =
* Theorem 1
  + If
  + Then
    - This holds for any optimization problem with convex constraints
  + Corollary
* Remarks on Theorem 1
  + Application does not require knowledge about the distribution
  + Places an upper bound on the k/N relation
  + Bound is not conservative
    - For a given e and B,
    - For a finite N, other terms are significant and uncertainty has more of an effect
  + For a conservative bound, the number of constraints actually removed must be less than e\*N
  + Removal is also a method of removing noise
* Theorem 2
  + If
  + Then with 1-B probability (at least)
    - v is a degradation margin
      * Shows a performance mismatch between the solution to the initial problem and the calculated solution
      * N and k increase as v reaches 0